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The role of thermocapillary instability in heat transfer in a liquid metal pool

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Abstract—During the treatment of metal with concentrated heat sources (plasma and laser welding, cutting etc.), high temperature gradients are created within the liquid metal pool. A capillary wave propagating along this pool carries non-uniformly heated metal out to the surface. Since the surface tension coefficient σ depends on temperature, surface forces arise, which are directed from these spots with low surface tension toward the spots with high σ. These forces can amplify the capillary wave, thus leading to instability. A simple expression for the increment of this instability for the molten pool of finite depth is obtained. It is shown that, for conditions of plasma arc welding, a wide spectrum of capillary waves exists in a liquid metal pool. The analysis of the non-linear phase of the instability shows the saturation of the waves' amplitude. The influence of the waves on the thermal conductivity and on the viscosity of the metal in the pool is estimated. It is shown that, for high energy flux densities, capillary waves might increase the effective thermal conductivity several times. Their influence upon the viscosity is even more pronounced. These estimations qualitatively agree with some recent data on heat and mass transfer intensity in the liquid metal during welding. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Since the end of the 1960s it has become evident that the thermocapillary effect (motion of a liquid non-uniformly heated over its surface) plays a significant role in heat and mass transfer in the liquid metal pool during metal welding and cutting. The surface tension coefficient of liquid metal, as of any other liquid, depends on temperature. If the surface of a liquid metal is heated non-uniformly, the surface tension coefficient σ varies along the surface. This gives rise to a surface force

$$F_s = \frac{d\sigma}{dx} = \frac{d\sigma}{dT}\frac{dT}{dx} \tag{1}$$

which is directed from locations with a lower σ towards those with higher surface tension coefficients. This force induces the motion of the liquid along the surface. When metal is treated with concentrated intensive heat fluxes (plasma or laser welding, cutting etc.), there are high temperature gradients along the surface, which create powerful surface forces. As a rule, σ falls with a temperature increase, therefore, the surface force drives the metal from the center toward the periphery of the liquid metal pool. This results in a pressure rise at the pool periphery, which forces the metal there to dive and move along the bottom of the pool toward the center, where the metal rises and completes the vortex. In the case of a positive $d\sigma/dT$ value, the direction of the vortex is reversed. It is believed that this change in the vortex direction is responsible for a dramatic change in weld penetration and weld quality [1, 2].

The processes of mass and heat transfer in the liquid metal pool are substantially intensified by the metal flows induced by the surface tension variation. Calculations of such flows are carried out in many works; see for examples refs. [3, 4] or review [5]. In these works it is assumed that the metal flow is stable and that there is no small scale activity in the pool. In this paper we would like to emphasize that, for conditions typical for welding, the liquid metal pool might be unstable. A wide spectrum of capillary microwaves might develop in the pool. This turbulization of the pool might additionally intensify heat and mass transfer significantly, thus increasing the effective thermal conductivity and viscosity.

The instability we are referring to is connected with the metal temperature variation in the direction perpendicular to the surface. This temperature variation always exists in the liquid metal pool, in many cases it is even more pronounced compared to the variation parallel to the surface. By itself, this gradient does not produce any macro flows, however, it is known that this gradient can lead to thermocapillary instability (TCI) [6]. For the most common case $d\sigma/dT < 0$, the aperiodic instability of a liquid heated from below (from the depth) has been known since the 1950s [7]. In the case of the liquid metal pool heated from the surface, this aperiodic instability occurs if $d\sigma/dT > 0$. However, even if $d\sigma/dT < 0$ the pool heated from the surface might be unstable. A capillary wave propagating along the surface of the liquid, non-uniformly

NOMENCLATURE

A	wave amplitude at the surface
E	wave energy
$\dot{E}_s$	rate of wave energy dissipation due to surface force
$\dot{E}_v$	rate of wave energy dissipation due to viscosity
F_s	surface force
k	wave number
k_0	wave number separating stable and unstable parts of spectrum
k_{max}	wave number of the fastest growing wave
k_{min}	minimum wave number
H	pool depth
h	molten pool surface elevation
L	pool width
p	pressure
q	heat flux density
q_{cond}	heat flux density due to thermal conduction
q_{conv}	heat flux density due to convection
$T_0(z)$	undisturbed temperature profile
T_1	temperature perturbation
T_s	surface temperature
T_m	melting point
u	velocity of the melt in the pool
$\mathbf{V}(V_x, V_y, V_z)$	velocity vector.

Greek symbols

γ	complex wave number
κ	thermal conductivity
λ	wave length
ν	kinematic viscosity
ρ	density
σ	surface tension
$d\sigma/dT$	thermal coefficient of surface tension
τ	period of metal rotation in macro vortex
ϕ	velocity potential
χ	thermal diffusivity
ω	wave frequency
ω'	wave increment.

Subscripts

eff	effective value
0	undisturbed value
p	pulsing component
s	surface
v	viscosity.

heated through its depth, carries differentially heated layers to the surface. The surface forces which arise due to surface tension variations might amplify the capillary wave.

The possibility of instability setting in as oscillations (i.e. overstability) in a liquid non-uniformly heated through its depth was first mentioned by Takashima [8]. The problem was further considered by Gershuni and Zhukhovitskii [9], Garsia-Ybarra and Velarde [10], and Nepomnyaschii and Simanovskii [11]. It should be noted that the character of the instability depends on the heat transfer conditions at the surface. For the case of intense heat flux densities (10^3 W cm^{-2} and greater), the heat transfer rate does not significantly depend on the surface temperature (unless the metal starts to evaporate). Therefore, the consideration of refs. [9] and [11] (where stability of two immiscible liquid layers with a fixed temperature difference across these layers was examined) does not reflect the situation at hand. More general heat transfer conditions at the surface were considered in ref. [10]; however, this work treated the case of an infinitely deep pool, therefore, the most interesting case of wavelength comparable to the pool depth was not considered. Also, the main emphasis of this work was on how weak gravity influences the conditions of the onset of the instability (neutral conditions). However, gravity does not play any significant role in the liquid pool behavior, but because of the general character of the consideration in ref. [10], the dispersion relation obtained is complicated and difficult to analyze.

The present article consists of two parts. In the first part the linear analysis of the thermocapillary instability is performed for the conditions typical for the metal welding. The approach to the problem is close to that used in ref. [10], but an arbitrary pool depth–wavelength ratio is considered.

TCI can affect the heat and mass transfer in the pool only if the amplitudes of the waves are large enough. In order to estimate these amplitudes, the nonlinear phase of the instability should be considered, which to the best of our knowledge has not yet been done for TCI. In the second part of this article the possible scenario of the nonlinear phase of the instability is considered. Based on this scenario, the effective coefficients of thermal conductivity and viscosity are estimated.

The analysis presented below is made for plasma arc welding conditions. However, we believe that with some modifications it can be applied to other types of metal treatment with intense heat sources.

2. LINEAR ANALYSIS OF THE THERMOCAPILLARY INSTABILITY

Determination of the capillary wave stability includes the simultaneous consideration of the hydrodynamic processes together with heat transfer processes in the pool. Although the procedure of obtaining the dispersion relation is more or less standard and straightforward, the final expression is complex and difficult to analyze. It should be noted, however, that in almost all interesting cases, the TCI increment (decrement) is significantly smaller when compared to the capillary wave frequency, i.e. the relative change in the wave amplitude during the period is small. This allows one to simplify the consideration significantly—in the zero-order approximation, no processes leading to the wave amplitude change (viscosity and work by surface force) are included. After obtaining the velocity and temperature distributions, the work performed by viscosity and surface forces can be calculated. If the work done by the surface force exceeds the work performed by viscosity, the wave is unstable. The increment of the instability can be calculated as the difference between these two works related to the energy of the liquid motion due to oscillations.

Let us introduce the coordinate system with its z-axis directed out of the liquid, so that the plane $z = 0$ corresponds to an unperturbed surface and the bottom of the pool is located at $z = -H$. The pool surface is subjected to the heat flux of constant density q; the bottom of the pool is kept at the constant temperature T_m (the melting point temperature). This heat flux creates a temperature gradient directed perpendicular to the surface.

Following the program outlined above, let us calculate the velocity and temperature distribution in the absence of viscosity and surface tension variation. The motion of the liquid is described by the equations

$$\Delta\phi = 0 \tag{2}$$

$$\nabla\cdot\mathbf{V} = 0 \tag{3}$$

$$-\nabla p = \rho\,\partial\mathbf{V}/\partial t. \tag{4}$$

Here ϕ is the velocity potential, $\mathbf{V}(V_x, V_z)$ is the velocity vector, ρ is the density of the liquid, and p is the pressure. Since the viscosity is not taken into consideration, the boundary conditions are imposed on the normal component of the velocity V_z only. Namely, let h be the perturbed surface level, so that

$$h = \int V_z(z=0)\,\mathrm{d}t. \tag{5}$$

Then the pressure balance at the free surface is

$$\sigma\frac{\mathrm{d}^2h}{\mathrm{d}x^2} + p = 0. \tag{6}$$

The second boundary condition evidently is

$$V_z(z = -H) = 0. \tag{7}$$

The solution of the Laplace equation (2) having the form of the wave propagating in the x-axis direction is

$$V_x = A\frac{sh\{kz+kH\}}{sh(kH)}\exp(i\omega t + ikx) \tag{8}$$

$$V_z = iA\frac{ch\{kz+kH\}}{sh(kH)}\exp(i\omega t + ikx) \tag{9}$$

where A is the amplitude of the velocity oscillations at the surface, $k = 2\pi/\lambda$ is the wave number, and ω is the wave frequency. After substituting equation (8) into the boundary condition of equation (6), one obtains the well-known dispersion relation for the capillary wave propagating in the finite depth pool [12]:

$$\omega^2 = \frac{\sigma k^3}{\rho}th(kH). \tag{10}$$

We represent the temperature distribution as $T = T_0(z) + T_1(x, z, t)$, with T_0 satisfying boundary conditions $T_0(-H) = T_m$ (melting point) and $\mathrm{d}T_0(z=0)/\mathrm{d}z = -q/\kappa$, where q is the heat flux density and κ is the metal thermal conductivity. Now let us calculate T_1, the temperature perturbation caused by motion of the liquid. T_1 satisfies the equation

$$\frac{\partial T_1}{\partial t} - \chi\nabla^2T_1 = -V_z\frac{\mathrm{d}T_0}{\mathrm{d}z}, \tag{11}$$

where χ is thermal diffusivity, and $\mathrm{d}T_0/\mathrm{d}z$ is the stationary temperature gradient created by the heat flux at the surface. Boundary conditions for this equation are

$$T_1(z = -H) \tag{12}$$

$$\frac{\partial T_1(z=0)}{\partial z} = 0. \tag{13}$$

The condition of equation (13) reflects the fact that heat flux at the surface is fixed and does not depend on the surface temperature. The solution of equation (11), with boundary conditions of equations (12) and (13), is

$$T_1 = \left[\frac{sh[k(z+H)]}{sh(kH)} - \frac{k}{\gamma}\frac{cth(kH)}{ch(\gamma H)}sh[\gamma(z+H)]\right]\frac{iA}{\omega}\frac{\mathrm{d}T_0}{\mathrm{d}z}\exp(ikx) \tag{14}$$

where $\gamma^2 = k^2 + i\omega/\chi$. For simplicity, the factor $\exp(i\omega t)$ is hereafter omitted. For the surface temperature one has:

$$T_s = \frac{\mathrm{d}T_0}{\mathrm{d}z}h + T_1(z=0) = -i\frac{k}{\gamma}\frac{A}{\omega}\frac{\mathrm{d}T_0}{\mathrm{d}z}\frac{cth(kH)}{cth(\gamma H)}\exp(ikx) \tag{15}$$

and the surface force

$$F_s = \frac{\mathrm{d}\sigma}{\mathrm{d}x} = ikT_s\frac{\mathrm{d}\sigma}{\mathrm{d}T} = \frac{k^2}{\gamma}\frac{A}{\omega}\frac{\mathrm{d}T_0}{\mathrm{d}z}\frac{cth(kH)}{cth(\gamma H)}\exp(ikh). \tag{16}$$

After separating the real part we have

$$ReF_s = \frac{k^2}{\omega} A \frac{dT_0}{dz} \frac{d\sigma}{dT} cth(kH) \sqrt{\frac{\chi}{2\omega}}$$

$$\times \begin{bmatrix} \cos(kx) \dfrac{sh(a)ch(a)+\sin(a)\cos(a)}{\cos^2(a)ch^2(a)+\sin^2(a)sh^2(a)} - \\ \sin(kx) \dfrac{\sin(a)\cos(a)-sh(a)ch(a)}{\cos^2(a)ch^2(a)+\sin^2(a)sh^2(a)} \end{bmatrix} \quad (17)$$

where

$$a = H\sqrt{\frac{\omega}{2\chi}} = kH\sqrt{\frac{\omega}{2\nu k^2}\frac{\nu}{\chi}}. \quad (18)$$

From equation (9) one obtains

$$ReV_X(z=0) = -Acth(kH)\sin(kx). \quad (19)$$

One can see that the term proportional to $\sin(kx)$ in equation (17) is the only one which is in phase with velocity V_X and, therefore, participates in mechanical work. Performing the averaging of the work $\dot{E}_s = V_x F_s$ along the wave, and taking into account the fact that $\overline{\sin^2(kx)} = 1/2$, we have

$$\dot{E}_s = -\frac{A^2}{2} cth(kH) \frac{k^2}{\omega^2} \frac{dT_0}{dz} \frac{d\sigma}{dT} \sqrt{\frac{\chi}{2\omega}} f(a) \quad (20)$$

where

$$f(a) = \frac{sh(a)ch(a)-\sin(a)\cos(a)}{\cos^2(a)ch^2(a)+\sin^2(a)sh^2(a)}. \quad (21)$$

One can see that if $dT_0/dz \cdot d\sigma/dT < 0$, the work is positive and instability might occur.

The mechanism of the instability can be illustrated as follows: heat transfers in the capillary wave by means of conduction and convection. The motion of the nonuniformly heated liquid to the surface creates the heat flux $q_{conv.}$. Since the total heat flux to the surface is fixed, the equal and opposite directed heat flux due to thermal conduction arises, $q_{cond.}$. Although both fluxes compensate each other, $q_{cond.} + q_{conv.} = 0$, the temperatures created by them do not compensate each other (the difference is the surface temperature variation). This is because the phase relations are different for each of these heat transfer mechanisms. In the case of heat convection, the phase shift between heat flux and temperature is either 0 or π depending on the dT_0/dz sign. This means that the phase shift between V_x and the corresponding temperature gradient along the surface dT/dx is one-quarter of the period and, therefore, the corresponding surface force does not produce any average work. On the other hand, for heat transfer due to conduction, the heat flux to the surface and the surface temperature are shifted from each other by $\pi/4$ [13]. The corresponding surface force (proportional to $d\sigma/dT \cdot dT/dx$) is shifted from V_x by one-eighth of the period and, therefore, does produce mechanical work. These phase relationships are shown in Fig. 1.

Returning to the increment evaluation, let us calculate the loss of the wave energy due to viscosity. For this loss per unit surface area per unit time, one has [12]

$$\dot{E}_v = \frac{\rho\nu}{2}\int_{-H}^{0} \overline{\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2} dz = A^2 k^2 \rho\nu \frac{sh(2kH)}{sh^2(kH)}. \quad (22)$$

Now let us calculate the energy of the wave. Taking into account that according to the virial theorem, the total energy is the kinetic energy doubled, one has

$$E = 2E_{kin} = \rho \int_{-H}^{0} (\overline{v_x^2} + \overline{v_z^2})\, dz = \frac{\rho A^2}{4k} \frac{sh(2kH)}{sh^2(kH)}. \quad (23)$$

For the amplitude increment ω' (half of the energy increment), one has, finally,

$$\omega' = \frac{\dot{E}_v + \dot{E}_s}{2E} = -2\nu k^2 - \frac{k^3}{2\rho\omega}\sqrt{\frac{\chi}{2\omega}} \frac{dT_0}{dz} \frac{d\sigma}{dT} cth(kH) f(a). \quad (24)$$

When $a \gg 1$ and $kH \gg 1$, this expression reduces to

$$\omega' = -2\nu k^2 - \frac{k^3}{2\rho\omega}\sqrt{\frac{\chi}{2\omega}} \frac{dT_0}{dz} \frac{d\sigma}{dT} \quad (25)$$

which can be obtained by neglecting gravity in the formulae [10].

It is convenient to start the TCI analysis from the case of an infinitely deep pool ($kH \gg 1$). The first term in equation (25) is proportional to k^2 and the second, see equation (10), is proportional to $k^{3/4}$. Therefore, for very long waves ($k \to 0$), the stabilization effect of viscosity is negligible and the waves are unstable. When the wave number k increases, the increment ω' rises, reaches the maximum, and then changes sign. The threshold wave number which corresponds to the neutral situation ($\omega' = 0$), according to equations (10) and (25), is

$$k_0 = \left[\frac{1}{4\rho\nu}\left(\frac{\rho}{\sigma}\right)^{3/4}\left(\frac{\chi}{2}\right)^{1/2}\frac{dT_0}{dz}\left(-\frac{d\sigma}{dT}\right)\right]^{4/5}. \quad (26)$$

The wave number which corresponds to the maximum increment is

$$k_{max} = (3/8)^{4/5} k_0 \approx 0.46 k_0. \quad (27)$$

For the maximum increment one has

$$\omega'(k_{max}) = 2\nu k_o^2[(\tfrac{3}{8})^{3/5} - (\tfrac{3}{8})^{8/5}] \approx 0.69\nu k_0^2 \quad (28)$$

and the frequency of the capillary wave corresponding to this wave number is

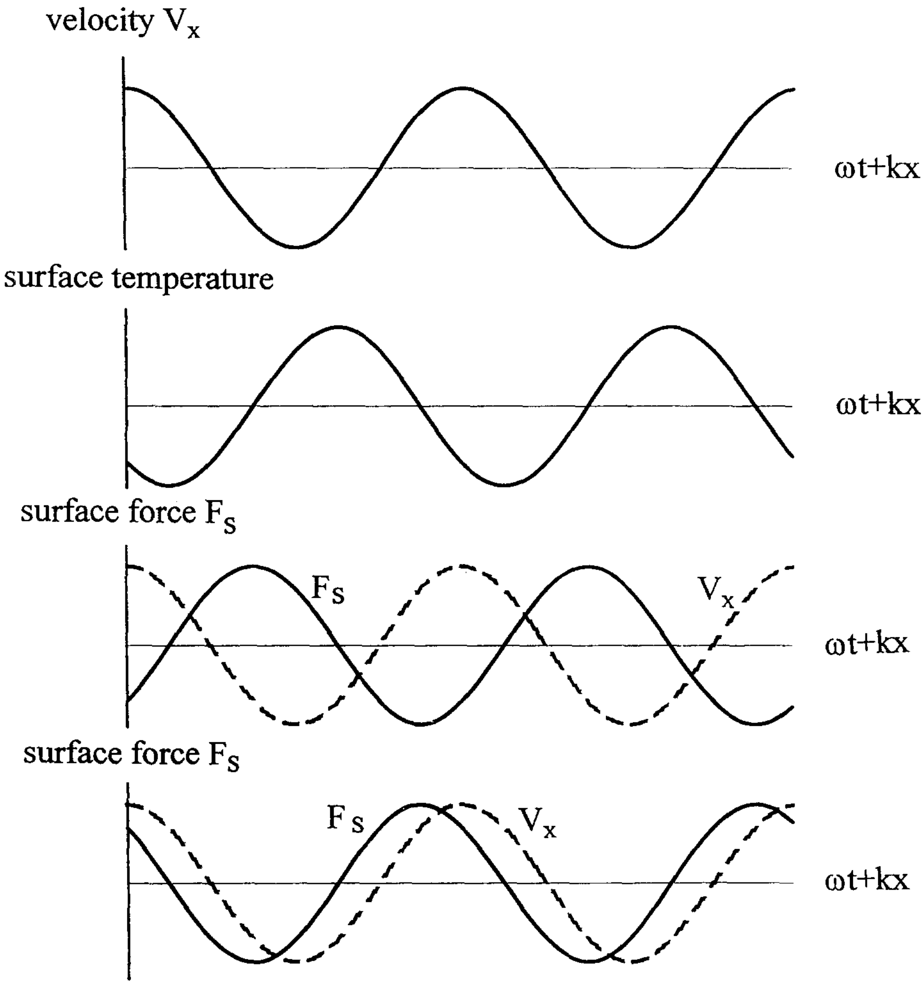

Fig. 1. Phase relationship in a capillary wave: (a) velocity along the surface, (b) surface temperature, (c) surface force (stable case), (d) surface force (unstable case).

$$\omega(k_{\max}) = \left(\frac{3}{8}\right)^{6/5} \sqrt{\frac{\sigma k_0^3}{\rho}} \approx 0.31 \sqrt{\frac{\sigma k_0^3}{\rho}}. \qquad (29)$$

For a pool of finite depth the increment is higher and the frequency of oscillations is lower compared to these parameters for the pool of infinite depth. The difference between these two cases arises when parameter kH approaches unity. Since $\omega \gg 2\nu k^2$ (decrement due to viscosity is much less than frequency), note that, when $kH < 1$, it is still possible for parameter a to be bigger or smaller than unity. One can see from equations (10) and (24) that if $kH < 1$ but $a > 1$ then

$$\omega' = -\sqrt{\frac{\chi}{8}} \frac{1}{\rho k H} \left(\frac{\rho}{\sigma H}\right)^{3/4} \frac{dT_0}{dz} \frac{d\sigma}{dT} \qquad (30)$$

i.e. the increment ω' increases when k and H decrease. For even smaller values of k and H, when both $kH < 1$ and $a < 1$, the increment becomes

$$\omega' = -\frac{1}{6\rho\chi} (kH)^2 \frac{dT_0}{dz} \frac{d\sigma}{dT} \qquad (31)$$

i.e. the increment decreases with the further decrease of k and H. This means that there is a specific wave number (except $k_{\max}$) and a specific pool depth for which the increment has a maximum value.

3. TCI ANALYSIS FOR WELDING POOL CONDITIONS

The dependencies of $k_{\max}$, $\omega'(k_{\max})$ and $\omega(k_{\max})$ on the surface heat flux density $q = \kappa\, dT_0/dz$ are shown in Fig. 2. Thermal conductivity κ and other liquid metal parameters (for liquid steel, [14]) are listed in Table 1.

In an infinitely wide pool, those waves sufficiently long ($k < k_0$) are always unstable. In reality, the finite width of the pool imposes a limit on the minimum possible wave number. As a rule, the pool width L does not exceed 1 cm, so that $k > k_{\min} \sim 10\ \text{cm}^{-1}$. Only these waves are considered further. From equation (26), see also Fig. 2, one can see that threshold wave number k_0 increases with an increase in the heat flux density, and that a wide spectrum of capillary

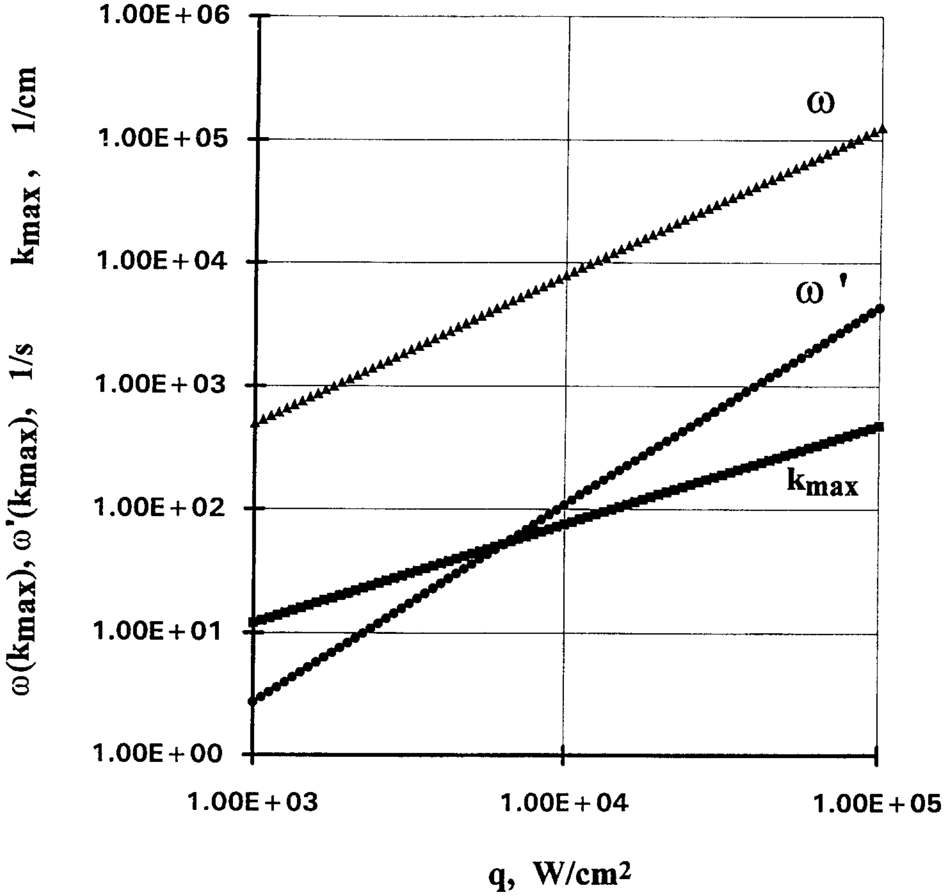

Fig. 2. Wave number k_{max} frequency $\omega(k_{max})$, and increment $\omega'(k_{max})$ of the fastest growing capillary wave vs heat flux density.

Table 1. Liquid parameters used in calculations

Parameter	Value	Reference
Density [g cm^{-3}]	7.0	[14]
Thermal conductivity [W cm^{-1} K^{-1}]	0.15	[14]
Specific heat [J g^{-1} K^{-1}]	0.79	[14]
Kinematic viscosity [cm^2 s^{-1}]	6×10^{-3}	[14]
Surface tension [N m^1]	1.2	[5]
Temperature coefficient of surface tension [N m^{-1} K^{-1}]	-0.5×10^{-3}	[5]

waves are unstable at high q values. On the other hand, when q is relatively small (does not exceed $\sim 10^3$ W cm^{-2}), the threshold wave number k_0 is less than the minimum possible wave number k_{min}. TCI does not develop under these conditions.

As mentioned in the Introduction, there is motion of the liquid metal along the pool surface (macro vortex) which is due to the temperature gradient parallel to the surface.† This motion itself does not affect the TCI but it puts a limit on τ, the time during which any particular part of the liquid experiences the TCI. This time can be estimated as L/u, where L is the pool width and u is the velocity of the liquid along the surface. Calculations and measurements give $u < 10$ cm s^{-1} [17]. Since $L < 1$ cm, we have $\tau \sim 0.1$ s. For TCI to develop, the product $\omega'(k_{max})\tau$ must exceed unity significantly, such as $\omega'(k_{max})\tau > 5$–10. From Fig. 2 one can see that this occurs when $q > 10^4$ W cm^{-2}. Therefore, for open arc welding ($q < 10^4$ W cm^{-2} [18]), capillary waves, being unstable, do not have enough time to develop. Plasma arc welding and laser welding are characterized by higher q values [16]. For these types of welding this limitation does not exist.

For the conditions typical for plasma arc welding ($q = 10^4$ W cm^{-2} for liquid steel), the increment dependence on wave number for some pool depths is depicted in Fig. 3. In the same figure the ω'/ω ratio is shown, whose small value permits the applicability of

† In this macro vortex (Marangoni vortex), metal moves along the surface within a boundary layer of thickness $l_{mar} \sim 10^{-3}$ cm [15, 16] and returns back throughout the whole bulk of the pool: the pool is covered with a 'skin' of a high velocity gradient across it. Although the uniform velocity distribution dues not change the above consideration, the distribution with a steep velocity gradient does. However, for interesting wave numbers (less than $1/l_{mar} \sim 10^3$ cm^{-1}), this 'skin' can be considered as rigid and its existence does not influence TCI development.

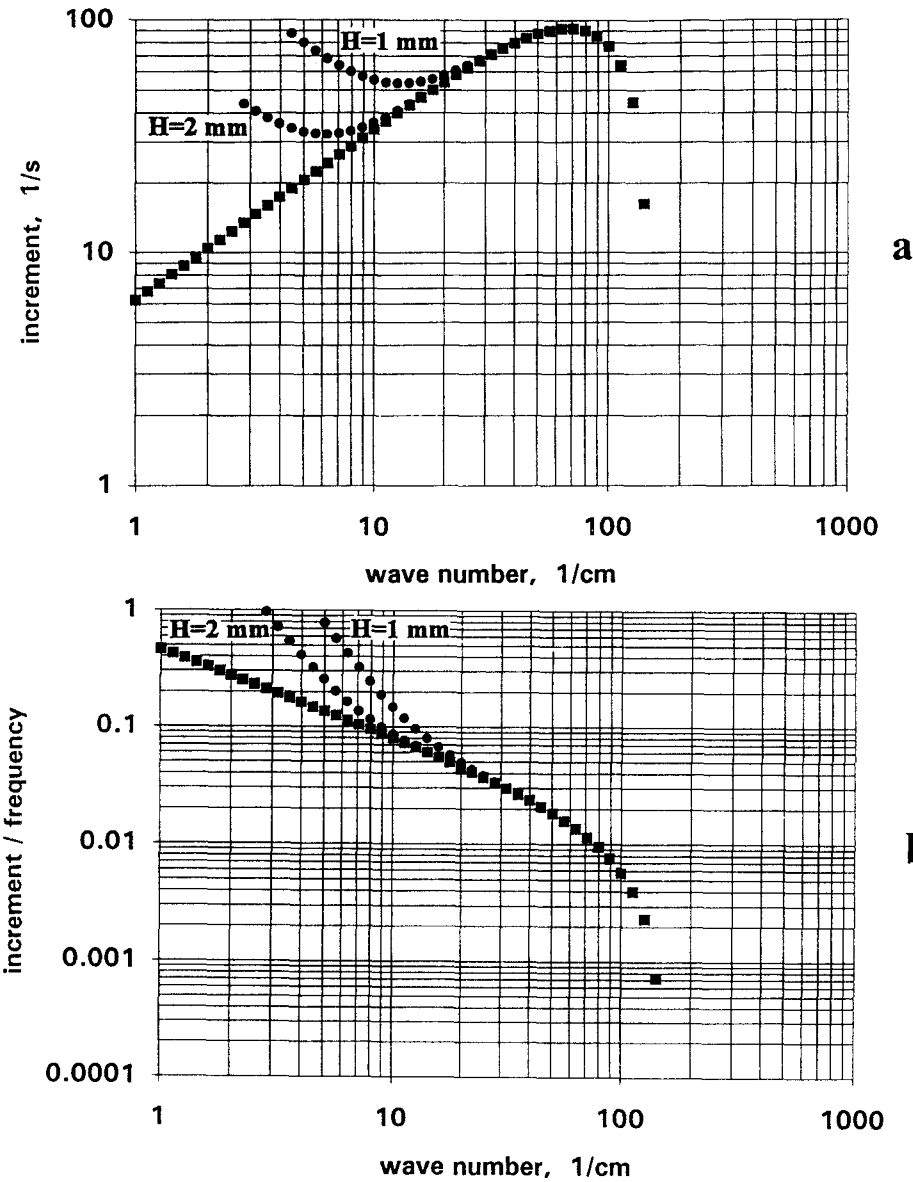

Fig. 3. Increment (a) and increment–frequency ratio (b) vs wave number for two pool thicknesses ($H = 1$ mm, 2 mm, circles) and for an infinitely deep pool (squares).

the method used. One can see that condition $\omega'/\omega < 1$ is satisfied when $k > 3$ cm^{-1}. Therefore, for waves existing in a welding pool ($k > k_{\min} \sim 10$ cm^{-1}), the method used is applicable, although for the longest possible waves the condition $\omega'/\omega < 1$ is satisfied only marginally (the instability is close to an aperiodic one). Note that because of the wave number limitation $k > k_{\min}$, the part of $\omega'(k)$ dependence, where the increment experiences the maximum value, is not realized in the conditions typical for welding. For these conditions the fastest growing oscillations are those with the $k_{\max}$ wave number.

Another conclusion which can be drawn from Fig. 3 is that although calculations were carried out for kH close to or even equal to unity, the increments for finite and infinite depth pools do not show a drastic difference, so that when kH is not less than unity, the formulae for an infinitely deep pool can be used.

4. NONLINEAR STABILIZATION DUE TO THERMAL CONDUCTIVITY AND VISCOSITY INCREASE

During the first (linear) phase of instability, the wave with the highest increment $\omega'(k_{\max})$ is the most excited in the pool. As for any capillary wave, this wave agitates the liquid within the near-surface layer of $1/k_{\max}$ thickness. This motion creates pulsing convective heat and momentum transfer, i.e. it leads to additional (pulsing) components of thermal conductivity κ_p and viscosity ν_p. The order of magnitude of these parameters is

$$\chi_p \sim \nu_p \sim hA \sim A^2(t)/\omega \qquad (32)$$

where $\chi_p = \kappa_p/\rho C$ and ν_p are pulsing components of thermal diffusivity and kinematic viscosity, and h and A, as before, are the amplitudes of the surface level and the velocity oscillations, respectively. Under the constant heat density flux condition, the rise of the thermal conductivity leads to the decrease in the temperature gradient in the near-surface region. This diminishes the main force which drives the thermocapillary instability and decreases its increment. The higher the wave amplitude, the higher the effective thermal conductivity; the lower the temperature gradient, the smaller the instability increment. An additional increment reduction is due to the increase in the viscosity. The amplitude of the wave increases until its increment becomes zero. The temperature distribution at this phase of the process is depicted in Fig. 4; at a depth below approximately $1/k_{max}$ the temperature gradient is small; farther from the surface it restores its undisturbed value dT_0/dz. From equation (24) one can see that shorter waves need less thermal conductivity increase to be stabilized; the temperature gradient reduction which stabilizes waves with $k = k_{max}$ also stabilizes shorter waves. However, for oscillations with bigger wavelengths, the effective temperature gradient (averaged over the wavelength) is still large. This means that while the initially most dangerous mode stops rising, the waves with bigger wavelengths (smaller wave numbers) remain unstable. The amplitude of these waves increases until they also stop growing. The process of nonlinear stabilization of thermocapillary instability can be described as the movement of the boundary wave number k_0, which separates the already stable from still growing waves, toward the smaller wave numbers.

As a result of the described process, a wide spectrum of capillary waves develops in the layer. These oscillations increase the effective thermal conductivity and viscosity of the liquid. To estimate the order of magnitudes of these parameters, suppose that at some point in time the boundary wave number, separating stationary and still growing waves, is k_b. Let us accept the simplest approximation of the temperature dis-

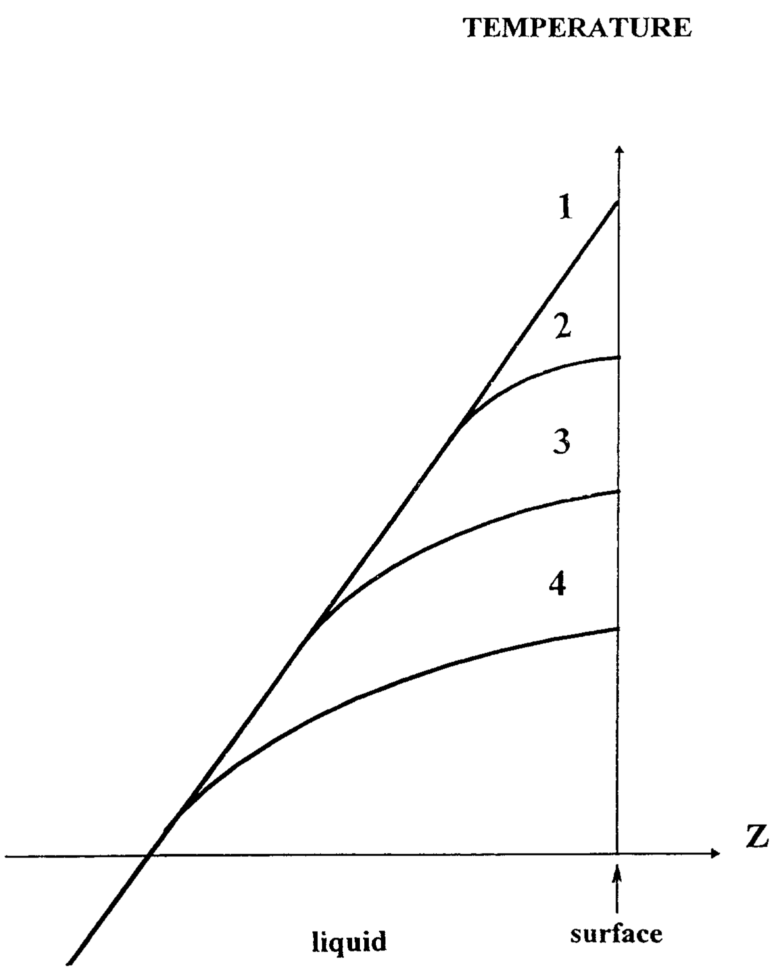

Fig. 4. Temperature distribution in the pool at different times: $t_4 > t_3 > t_2 > t_1$. t_1 corresponds to the initial distribution.

tribution; at distances from the surface greater than $1/k_b$, the temperature gradient has not yet changed and, therefore, is equal to the undisturbed value dT_0/dz. At distances smaller than $1/k_b$, the temperature gradient is constant and equal to the value which provides the zero increment of the wave with k_b wave number. With the given thermal flux density q, the temperature gradient is inversely proportional to the effective thermal conductivity:

$$\frac{dT}{dz}(k_b) = \frac{q}{\kappa_{eff}(k_b)} = \frac{q}{\rho c \chi_{eff}(k_b)} \quad (33)$$

here $\kappa_{eff}(k_b)$ and $\chi_{eff}(k_b)$ are the effective thermal conductivity and thermal diffusivity of the liquid layer, respectively, and c is the thermal capacity. The argument k_b is specified in order to emphasize that k_{eff} and χ_{eff} are referred to at the moment when the boundary wave number is k_b. The effective thermal diffusivity and viscosity are equal to $\chi_{eff} = \chi_0 + \chi_p$ and $\nu_{eff} = \nu_0 + \nu_p$, where index '0' designates unperturbed components. Both χ_p and ν_p have the same order of magnitude, and for the sake of simplicity we assume $\chi_p = \nu_p$.

The condition for the increment of the wave with the wave number k_b to vanish is

$$4\rho[\nu_0 + \chi_p(k_b)] = \frac{k_b}{\omega(k_b)}\sqrt{\frac{\chi_0 + \chi_p(k_b)}{2\omega(k_b)}} \times \frac{d\sigma}{dT}\frac{q}{\rho c[\chi_0 + \chi_p(k_b)]} cth(k_b H) f(a). \quad (34)$$

This is the equation used to calculate $\chi_p(k_b)$. The solutions of this equation obtained using parameters for liquid steel are shown in Fig. 5. This figure depicts relative increases of viscosity $\nu_p(k_b)/\nu_0$ and thermal diffusivity $\chi_p(k_b)/\chi_0$ as a function of the boundary wave number k_b with several fixed heat flux densities.

One can see that the smaller the k_b, the more the thermal conductivity and viscosity increase. The most interesting case is the case of a long wave (kH of order equal to less than unity), when the wave motion involves the liquid in the entire pool, not just the near-surface layer. Let k_b correspond to a capillary wave with $\lambda = L/2$, the pool width is 1 cm, so that $k_b = 12$ cm^{-1}. From Fig. 5 one can see that the pulsing motion of the liquid, due to development of thermocapillary instability, might increase the intensity of heat and mass transfer significantly. The higher the heat flux density, the bigger the increase in thermal conductivity and viscosity. Even for conditions of arc welding with its relatively low heat fluxes, the effect is more than noticeable. From Fig. 5 we obtain for $q = 10^4$ W cm^{-2}, $\chi_p/\chi_0 = 2$–3 and $\nu_p(k)/\nu_0 = 10$–13. The increase in viscosity is more pronounced, which obviously results from the low Prandtl number of liquid steel. For higher heat flux densities, both coefficients rise more substantially; for $q = 10^5$ W cm^{-2} one has $\chi_{eff}/\chi_0 = 13$ and $\nu_{eff}/\nu_0 = 56$.

5. COMPARISON WITH RECENT DATA ON HEAT AND MASS TRANSFER ENHANCEMENT

Since direct measurements of χ and ν of the liquid metal are not possible during welding, indirect indications should be used. In their article, ref. [19], Choo and Szekely made a critical analysis of the reason for the discrepancy between observed and calculated welding pool shapes. Calculations [20] have shown a deep and narrow pool, whereas the experiments [16] have shown the welding pool to be close to hemispherical. Choo and Szekely concluded that there is a turbulent motion in a pool, which intensifies heat and mass transfer. According to their estimations, the turbulent viscosity should be 30 times as much as laminar viscosity, in order to bring calculations into conformity with observations.

Waszink and Van Den Heuvel [21] considered heat transfer in the molten droplet hanging at the tip of the GMAW electrode. They pointed out that laminar (conductive) thermal conductivity is unable to provide a heat transfer rate high enough to explain the observed speeds of electrode melting at the solid–liquid interface. According to their estimations, there is a mismatch of the order of 10–20 times. Authors of ref. [21] suggested a strong convective component of thermal conductivity and attribute the convective motion to the action of thermocapillary or electromagnetic forces.

Jones *et al.* [22] considered free oscillations of the shape of such a hanging droplet. They calculated the frequency and damping rate of these oscillations. It turned out that the observed damping rate of these oscillations exceeds the calculated rate by many times. They concluded that this mismatch can be eliminated by assuming that the viscosity of liquid steel within the droplet is approximately 60 times its table value.

We conclude that although the estimated values of χ_{eff}/χ_0 and ν_{eff}/ν_0 look rather large, there are some indications that such a large heat and mass transfer enhancement really exists in welding conditions.

6. REMARK ON WAVE–WAVE INTERACTION

The described scenario is a quasi-linear one. The formula obtained in the linear approximation is used to calculate the wave increment. This approach is valid if the nondimensional wave amplitude is small, i.e. if $h/\lambda \ll 1$. From equation (33) one has

$$\frac{h}{\lambda} \sim \frac{\sqrt{\chi_p/\omega}}{\lambda}. \quad (35)$$

For a liquid-steel pool $L = 1$ cm, the nondimensional amplitudes at which waves with $k_b = L/2$ stabilize are: 2.1×10^{-2} ($q = 10^4$ W cm^{-2}), 3.1×10^{-2} ($q = 3 \times 10^4$ W cm^{-2}) and 4.8×10^{-2} ($q = 10^5$ W cm^{-2}). We see that the condition of quasi-linear approximation applicability is satisfied for heat flux densities within the 10^4–10^5 W cm^{-2} range.

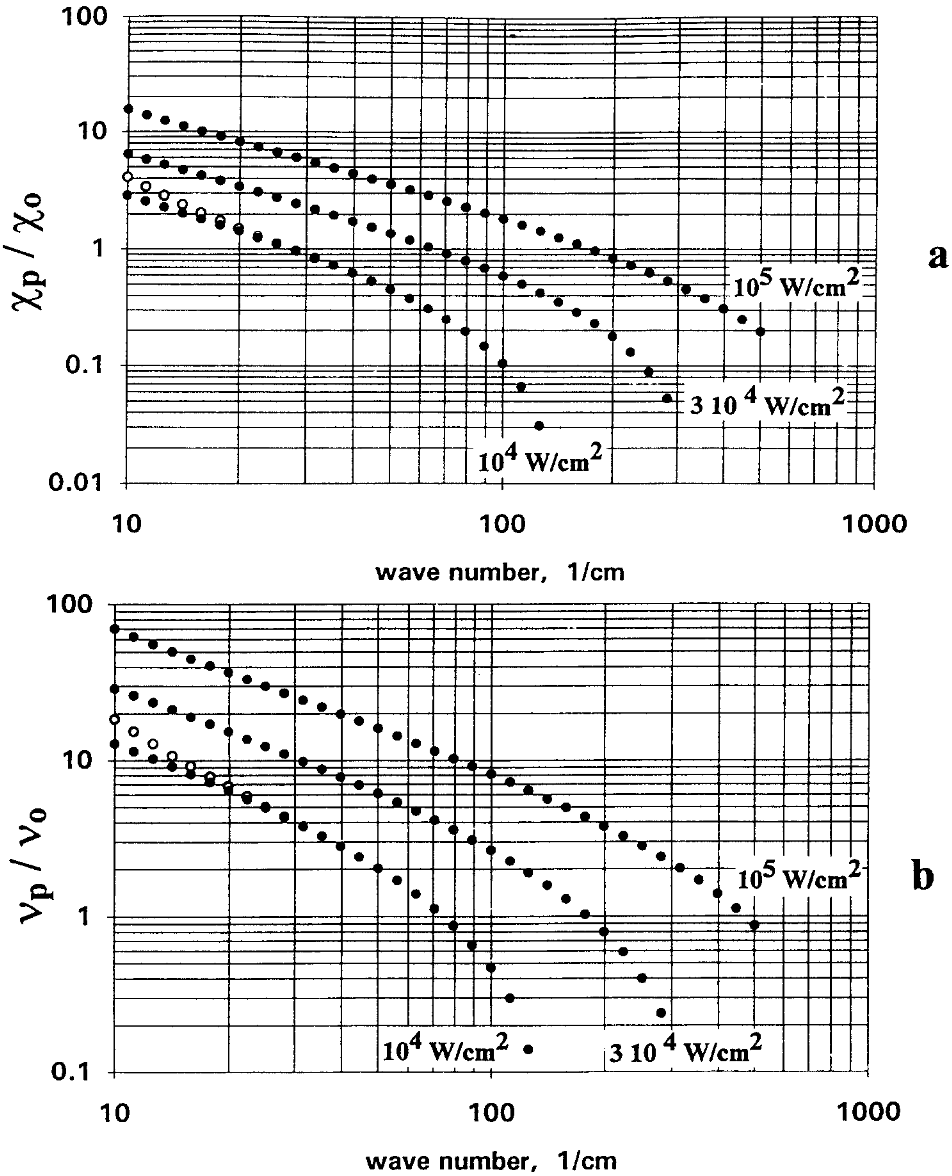

Fig. 5. Pulsing components of thermal diffusivity (a) and viscosity (b) vs boundary wave number at different heat flux densities. For $q = 10^4$ W cm^{-2}, curves for $H = 1$ mm (open circles) and $H = 1$ cm (filled circles) are shown.

At higher heat flux densities, the wave amplitude rises, so that other mechanisms of nonlinear stabilization should be considered. Since the dispersion relation $\omega(k)$ is of a decay type, it is likely that this mechanism is the confluence of two unstable waves into one, so that the resulting wave damps quickly:

$$\omega(k_1)+\omega(k_2) = \omega(k) \quad k_1+k_2 = k$$

$$k_1, k_2 < k_0 \quad k > k_0.$$

However, the mechanism of the nonlinear stabilization at higher heat flux densities demands special considerations which are out of the scope of this article.

7. CONCLUSION

In this paper:

(1) The capillary wave instability in the liquid metal pool, subjected to the intensive heat flux on its surface, is considered (thermocapillary instability, TCI). During this instability, the motion of near-surface layers is developed.

(2) The liquid layer (pool) of finite thickness is considered. By calculating the work performed by the surface force (due to surface tension variations) and viscosity, the expression for the TCI increment is obtained. The increment grows as heat flux density q grows; with a given q, at first the increment increases, then falls with a decrease in the pool depth.

(3) The non-linear phase of the instability is considered. It is suggested that TCI is stabilized due to the rise of the turbulent (pulsing) components of the viscosity and thermal conductivity. Based on this hypothesis, the equation to calculate the stationary values of these parameters is obtained.

The estimations were carried out for a liquid steel pool. It was shown that for heat flux densities $q > 10^3$ W cm^{-2}, the welding pool is unstable (TCI has a positive increment). However, due to macroscopic movement of the liquid in the pool, the TCI does not have enough time to develop unless q exceeds 10^4 W cm^{-2}.

As a result of TCI nonlinear development, the effective values of viscosity and thermal conductivity can rise significantly. For example, for $q = 3 \times 10^4$ W cm^{-2}, estimations give a six-fold increase in thermal conductivity, and a 25-fold increase in viscosity. The authors of refs. [19], [21] and [22] came up with close numbers while comparing observed and calculated intensities of heat and mass transfer in the welding pool and the molten droplet hanging at the tip of the welding electrode.

When the heat flux density is not too high, $q < 10^5$ W cm^{-2}, the amplitude of oscillation of the surface level remains much less than the wavelength (the non-dimensional amplitude of the wave is much less than unity). With these q values, the wave–wave interaction does not participate in TCI stabilization. A further increase in the heat flux density leads to a more significant role of wave fusion in TCI stabilization.

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